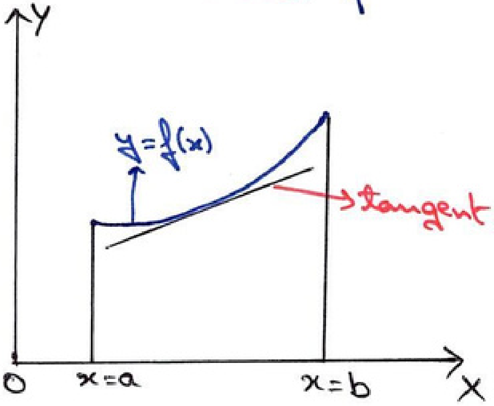
CURVE TRACING

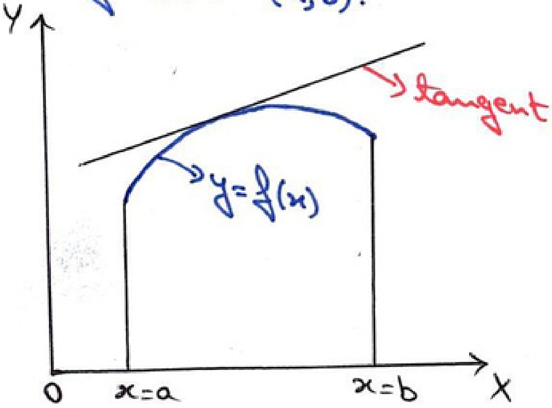
CONCAVE UPWARD :-

Let $y = f(x)$ be a continuous curve/function defined in (a,b). $f(x)$ is said to be concave upward (convex downwards) if all the points of the curve lies above any tangent to it in (a,b).



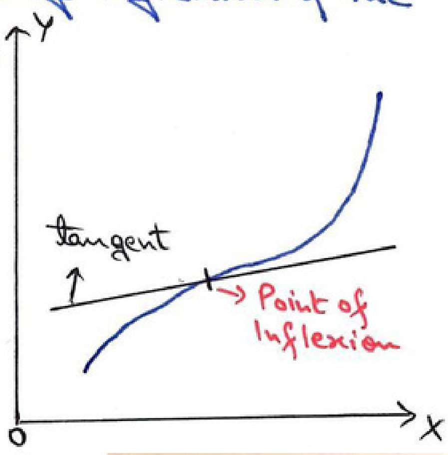
CONCAVE DOWNWARDS:-

Let $y = f(x)$ be a continuous function defined on (a,b). Then $f(x)$ is said to be concave downwards (convex upwards) if all the points of the curve lies below the tangent to ~~the~~ it on (a,b).



POINT OF INFLEXION :-

Let $y = f(x)$ be a continuous curve/function defined on (a,b). A point on the curve is said to be a point of inflexion if the curve passes through the tangent at that point. i.e. the curve crosses the tangent at a point, known as point of Inflexion.



METHOD TO FIND CONCAVITY & CONVEXITY OF A CURVE IN AN INTERVAL (a,b)

Let $y = f(x)$ be a given curve in (a, b).

Step I: find $\frac{dy}{dx}$.

Step II: find $\frac{d^2y}{dx^2}$

Step III: The curve will be concave upward in the interval (a,b) for which $\frac{d^2y}{dx^2} > 0$.

Step IV: The curve will be concave downward in the interval (a,b) for which $\frac{d^2y}{dx^2} < 0$.

METHOD TO FIND THE POINTS OF INFLEXION

Let $y = f(x)$ be a given curve defined in (a,b)

Step I:- find $\frac{dy}{dx}$.

Step II:- find $\frac{d^2y}{dx^2}$

Step III:- Equate $\frac{d^2y}{dx^2} = 0$ & find the values of 'x'.

Also; find the points $x = a, b, c, \ldots$ for which $\frac{d^3y}{dx^3} \neq 0$.

NOTE (1) If $x > a \Rightarrow x \in (a, \infty)$

(2) If $x \geq a \Rightarrow x \in [a, \infty)$

(3) If $x < a \Rightarrow x \in (-\infty, a)$

(4) If $x \leq a \Rightarrow x \in (-\infty, a]$

(5) If $|x| \leq a \Rightarrow -a \leq x \leq a$

(6) If $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$

(7) If $a > 0 \Rightarrow \frac{1}{a} > 0$

(8) If $a < 0 \Rightarrow \frac{1}{a} < 0$

ASYMPTOTE:- A straight line 'l' is called an asymptote of a curve iff the line lies on one side of curve & the perpendicular distance of a point P on that curve from the straight line 'l' tends to zero as point P approaches to infinity along that branch.

RECTANGULAR ASYMPTOTE:-

If an asymptote to a curve is either parallel to x-axis or parallel to y-axis, then it is called rectangular asymptote.

* **Horizontal Asymptote:-** An asymptote to a curve is said to be horizontal, if it is parallel to x-axis.

* **Vertical Asymptote:-** An asymptote to a curve is said to be vertical, if it is parallel to y-axis.

OBLIQUE ASYMPTOTE:-

An asymptote to a curve is said to be an oblique if it is neither parallel to x-axis nor parallel to y-axis.

METHODS TO FIND ASYMPTOTES

* **To find asymptotes parallel to x-axis.**

Let be the given equation of curve.

Equate, co-efficient of highest power of $x = 0$.

* **To find asymptotes parallel to y-axis**

Let $y = f(x)$ or $f(x,y) = 0$ be the given eqn. of curve.

Equate, co-efficient of highest power of $y = 0$

METHODS TO FIND OBLIQUE ASYMPTOTES

Let $f(x,y)=0$ be the given curve.

Step-I:- substitute $x=1$ & $y=m$ in n^{th} degree (highest) terms. & name it as $\phi_n(m)$.

Step-II: Evaluate $\phi_n(m)$; $\phi_{n-1}(m)$, & so on.

Step-III:- Equate $\phi_n(m)=0$ & find all the real roots.

Step-IV $\boxed{(a)}$ If roots are real & distinct, then $y=mx+c$ is the asymptote (oblique), where m is the root & 'c' can be evaluated using

(i) If $\phi'_n(m)\neq 0 \Rightarrow c\,\phi'_n(m)+\phi_{n-1}(m)=0$; $\phi'_n(m)\neq 0$.

(ii) If $\phi'_n(m)=0 \Rightarrow$ then there is no asymptote for this value of m.

(b) If roots are real & repeated, c can be evaluated using,

$$\frac{c^2}{2!}\phi''_n(m)+c\,\phi'_{n-1}(m)+\phi_{n-2}(m)=0 \quad ; \quad \phi''_n(m)\neq 0.$$

(for two repeating (equal) roots)

then $y=m_1x+c_1$ & $y=m_1x+c_2$ are the two parallel asymptotes to the given curve.

ASYMPTOTES FOR POLAR CURVES:-

Let $f(\theta)=\frac{1}{r}$ be a polar curve, then the asymptotes of the given curve is $r\sin(\theta-\alpha)=\frac{1}{f'(\alpha)}$; where α is the root of the equation $f(\theta)=0$.

NOTE:- If $y=mx+c$ is an oblique asymptote of the curve,

1) $\lim\limits_{x\to\pm\infty}\frac{y}{x}=m$ & $\lim\limits_{x\to\pm\infty}(y-mx)=c$

Q: Find the asymptotes of the curve

$$y = \frac{x^2+2x-1}{x}.$$

Soln:- The given eqn. of curve is $y = \frac{x^2+2x-1}{x}$

$$\Rightarrow \quad xy = x^2+2x-1$$

$$\Rightarrow \quad x^2-xy+2x-1=0 \quad \text{——— ①}$$

Asymptote parallel to x-axis,

Coefficient of highest power of x, ~~i.e. x^2~~ $=0 \Rightarrow 1$ which is constant. i.e. coeff. of $x^2 = 0$ $\Rightarrow$ $1=0$; which is not possible

$\therefore$ There is no asymptote parallel to x-axis.

Asymptote parallel to y-axis,

Coefficient of highest power of $y = 0$, ~~i.e.~~ $y = 0 \Rightarrow -x = 0$

$$\Rightarrow x = 0$$

$\therefore$ The asymptote parallel to y-axis is $x=0$.

Oblique asymptote

Method-I :- Let $y=mx+c$ be an oblique asymptote,

$$m = \lim_{x\to\infty} \frac{y}{x} = \lim_{x\to\infty} \frac{x^2+2x-1}{x^2}$$

$$= \lim_{x\to\infty} \left(\frac{x^2}{x^2}+\frac{2}{x}-\frac{1}{x^2}\right) = \lim_{x\to\infty}\left(1+\frac{2}{x}-\frac{1}{x^2}\right)$$

$$= 1+0-0 = 1.$$

$$c = \lim_{x\to\infty} (y-mx)$$

$$= \lim_{x\to\infty} \left\{\frac{x^2+2x-1}{x} - 1.x\right\}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}+2x-1-\cancel{x^2}}{x}$$

$$= \lim_{x\to\infty} \frac{2x-1}{x} = \lim_{x\to\infty} 2-\frac{1}{x} = 2$$

$\therefore$ $y = 1.x+2$, is the oblique asymptote.

Hence, the given curve ①, has two asymptotes,
$x=0$ & $y=x+2$

METHOD - II :- ~~substitute y=mx+c in eqn ①~~

The given eqn. of curve is

$$x^2 - xy + 2x - 1 = 0$$

highest$^{(2)}$ Coeff. of variables; (sub $x=1$ & $y=m$)

$$\phi_2(m) = 1-m \qquad \& \quad \phi_2'(m) = -1$$

highest$^{(1)}$ Coeff. of variable; (sub $x=1$ & $y=m$)

$$\phi_1(m) = 2$$

Constant term ; $\phi_0(m) = -1$

Now; $\phi_2(m) = 0$

$\Rightarrow 1-m=0$

$\Rightarrow \boxed{m=1}$; which is real & distinct,

Now; using $c\phi_2'(m) + \phi_1(m) = 0$

$c(-1) + 2 = 0$

$-c+2=0$

$\Rightarrow \boxed{c=2}$

$\therefore$ $y = mx+c$ is the reqd. oblique asymptote of ①

i.e. $\boxed{y = 1.x+2}$ is the reqd. oblique asymptote of ①

POINTS FOR TRACING A CURVE

Let $f(x,y)=0$ be the given curve.

1. **Symmetry**:- The curve will be

a) **Symmetric about x-axis** iff changing y to $-y$

$$\Rightarrow f(x,-y)=f(x,y).$$

b) **Symmetric about y-axis** iff changing x to $-x$

$$\Rightarrow f(-x,y)=f(x,y)$$

c) **Symmetric about origin** iff changing x to $-x$ & y to $-y$

$$\Rightarrow f(-x,-y)=f(x,y)$$

d) **Symmetric about $y=x$**, iff changing x to y

$$\Rightarrow f(y,x)=f(x,y)$$

e) **Symmetric about $y=-x$**; iff changing x to $-y$ & y to $-x$

$$\Rightarrow f(-y,-x)=f(x,y).$$

Q1: $x^3+y^3=3axy$; $a\geq 0$

Soln.: The given curve is $f(x,y)=x^3+y^3-3axy=0$. — ①

1). Changing x to $-x$

$$\Rightarrow f(-x,y)=(-x)^3+y^3-3a(-x)y$$
$$=-x^3+y^3+3axy \neq f(x,y)$$

2) changing y to $-y$

$$\Rightarrow f(x,-y)=x^3+(-y)^3-3ax(-y)$$
$$=x^3-y^3+3axy \neq f(x,y)$$

3) Changing x to $-x$ & y to $-y$

$$\Rightarrow f(-x,-y)=(-x)^3+(-y)^3-3a(-x)(-y)$$
$$=-x^3+(-y^3)-3axy$$
$$=-(x^3+y^3+3axy) \neq f(x,y)$$

4) Changing x to y

$$\Rightarrow f(y,x)=y^3+x^3-3ayx$$
$$=x^3+y^3-3axy=f(x,y)$$

5) changing ~~x~~ to $-y$ & y to $-x$

$$f(-y,-x) = (-y)^3 + (-x)^3 - 3a(-x)(-y)$$
$$= -y^3 - x^3 + 3axy$$
$$= -x^3 - y^3 + 3axy$$
$$= -(x^3 + y^3 - 3axy) \neq f(x,y).$$

$\therefore$ The given curve is only symmetric about $y=x$.

Q2: $a^2y^2 = x^2(a^2-x^2)$

Soln: The given curve is $f(x,y) = y^2 - \frac{x^2(a^2-x^2)}{a^2} = 0$ ①

1) Changing x to $-x$;

$$f(x,y) = y^2 - \frac{(-x)^2[a^2-(-x)^2]}{a^2} = y^2 - \frac{x^2(a^2-x^2)}{a^2} = f(x,y)$$

2) changing y to $-y$

$$f(x,-y) = (-y)^2 - \frac{x^2(a^2-x^2)}{a^2}$$
$$= y^2 - \frac{x^2(a^2-x^2)}{a^2} = f(x,y)$$

3) Changing x to $-x$ & y to $-y$

$$f(-x,-y) = (-y)^2 - \frac{(-x)^2[a^2-(-x)^2]}{a^2}$$
$$= y^2 - \frac{x^2(a^2-x^2)}{a^2} = f(x,y)$$

4) Changing x to y & y to x

$$f(y,x) = x^2 - \frac{y^2(a^2-y^2)}{a^2} \neq f(x,y)$$

5) Changing x to $-y$ & y to $-x$

$$f(-y,-x) = (-x)^2 - \frac{(-y)^2[a^2-(-y)^2]}{a^2}$$
$$= x^2 - \frac{y^2(a^2-y^2)}{a^2} \neq f(x,y)$$

$\therefore$ The curve is symm about x-axis, y-axis & origin.

Q3: $x(x^2+y^2) = a(x^2-y^2)$

Soln:- The given curve is $f(x,y) = x(x^2+y^2) - a(x^2-y^2) = 0$ — ①

1) changing x to -x;

$$f(-x,y) = (-x)[(-x)^2+y^2] - a[(-x)^2-y^2]$$
$$= (-x)(x^2+y^2) - a(x^2-y^2) \neq f(x,y)$$

2) Changing y to -y;

$$f(x,-y) = x[x^2+(-y)^2] - a[x^2-(-y)^2]$$
$$= x(x^2+y^2) - a(x^2-y^2) = f(x,y)$$

3) Changing x to -x & y to -y;

$$f(-x,-y) = (-x)[(-x)^2+(-y)^2] - a[(-x)^2-(-y)^2]$$
$$= -x[x^2+y^2] - a(x^2-y^2) \neq f(x,y)$$

4) changing x to y & y to x;

$$f(y,x) = y(y^2+x^2) - a(y^2-x^2) \neq f(x,y)$$

5) Changing x to -y & y to -x;

$$f(-y,-x) = (-y)[(-y)^2+(-x)^2] - a[(-y)^2-(-x)^2]$$
$$= -y(y^2+x^2) - a(y^2-x^2) \neq f(x,y)$$

∴ The curve is symmetric about x-axis only.

Q4: $y = x + \frac{1}{x}$

The given curve is $f(x,y) = y - x - \frac{1}{x} = 0$ — ①

1) changing x to -x;

$$f(-x,y) = y-(-x)-\frac{1}{(-x)} = y+x+\frac{1}{x} \neq f(x,y)$$

2) changing y to -y;

$$f(x,-y) = -y-x-\frac{1}{x} \neq f(x,y)$$

3) changing x to -x & y to -y;

$$f(-x,-y) = -y-(-x)-\left(-\frac{1}{x}\right) = -y+x+\frac{1}{x} = 0$$
$$\Rightarrow y-x-\frac{1}{x} = 0$$
$$\Rightarrow f(-x,-y) = f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x - y - \frac{1}{y} \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$f(-y,-x) =$ ~~$-y-(-x)-\frac{1}{(-x)} = -y+x+1$~~

$$= (-x) - (-y) - \left(\frac{1}{-y}\right)$$

$$= -x + y + \frac{1}{y} \neq f(x,y)$$

$\therefore$ The curve is symmetric about origin only.

Also; when x is negative; y is -ve
& when x is +ve; y is +ve

$\therefore$ The curve lies in I^st^ & III^rd^ quadrant only.

Q:5: $y^2(a-x) = x^3$; $a>0$

Soln: The given eqn. of curve is $f(x,y) = y^2(a-x) - x^3 = 0$ —①

1) changing x to $-x$;

$$f(-x,y) = y^2[a-(-x)] - (-x)^3$$
$$= y^2(a+x) + x^3 \neq f(x,y)$$

2) changing y to $-y$;

$$f(x,-y) = (-y)^2(a-x) - x^3$$
$$= y^2(a-x) - x^3 = f(x,y)$$

3) Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = (-y)^2[a-(-x)] - (-x)^3$$
$$= y^2(a+x) + x^3 \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x^2(a-y) - y^3 \neq f(x,y)$$

5) Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-x)^2[a-(-y)] - (-y)^3$$
$$= x^2(a+y) + y^3 \neq f(x,y)$$

$\therefore$ The given curve is symmetric about x-axis only.

Q6: $y = \frac{x^2+1}{x+1}$

Soln: The given curve is $f(x,y) = y - \frac{x^2+1}{x+1} = 0$ — ①

1) Changing x to $-x$;

$$f(-x,y) = y - \frac{(-x)^2+1}{(-x)+1} = y - \frac{x^2+1}{-x+1} \neq f(x,y)$$

2) Changing y to $-y$;

$$f(x,-y) = -y - \frac{x^2+1}{x+1} \neq f(x,y)$$

3) Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = -y - \frac{(-x)^2+1}{(-x)+1} = 0$$

$$\Rightarrow -y^2 + \frac{x^2+1}{x-1} \neq f(x,y)$$

$$\Rightarrow \cancel{y^2 = \frac{x^2+1}{x+1}}$$

4) Changing x to y & y to x;

$$f(y,x) = x - \frac{y^2+1}{y+1} \neq f(x,y)$$

5) Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = \cancel{-y}\ -x - \frac{(-y)^2+1}{(-y)+1}$$

$$= -x - \frac{y^2+1}{-y+1} \neq f(x,y)$$

∴ The curve is not symmetric about any line or axis.

Q7: $y = \frac{x^3+1}{x}$

Soln:- The given curve is; $f(x,y) = xy - x^3 - 1 = 0$ — ①

1) Changing x to $-x$;

$$f(-x,y) = (-x)y - (-x)^3 - 1$$

$$= -xy + x^3 - 1 \neq f(x,y)$$

2) Changing y to $-y$;

$$f(x,-y) = x(-y) - x^3 - 1$$

$$= -xy - x^3 - 1 \neq f(x,y)$$

3). Changing x to $-x$, y to $-y$;

$$f(-x,-y) = (-x)(-y) - (-x)^3 - 1$$
$$= xy + x^3 - 1 \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = (y)(x) - y^3 - 1$$
$$= yx - y^3 - 1 \neq f(x,y)$$

5) Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-y)(-x) - (-y)^3 - 1$$
$$= xy + y - 1 \neq f(x,y)$$

∴ The given curve is not symmetric about any line or axis.

Q8: $y = \dfrac{x^3}{1+x^2}$.

Soln:- The given curve is; $f(x,y) = y(1+x^2) - x^3 = 0$

$$f(x,y) = y + x^2y - x^3 = 0 \quad \text{— ①}$$

1) Changing x to $-x$;

$$f(-x,y) = y + (-x)^2y - (-x)^3$$
$$= y + x^2y + x^3 \neq f(x,y)$$

2) Changing y to $-y$;

$$f(x,-y) = (-y) + x^2(-y) - x^3$$
$$= -y - x^2y - x^3 \neq f(x,y)$$

3) Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = (-y) + (-x)^2(-y) - (-x)^3$$
$$= -y - x^2y + x^3 \cancel{\neq} = \overset{0}{\cancel{f(x,y)}} \Rightarrow f(-x,-y) = f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x + y^2x - y^3 \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = -\cancel{x} + (-x)^2(-y)^2 - (-\cancel{y}^3)^3 = -\cancel{x} - x^2y + \cancel{y}^3 \neq f(x,y)$$

Also; when x is -ve, y is -ve

& when x is +ve, y is +ve.

∴ The curve is ~~not~~ symmetric about ~~any~~ origin line & axis but lies in 1st & 3rd quadrant.

Q9: $y = \frac{x^3-3}{2x-4}$

The given curve is; $f(x,y) = y - \frac{x^3-3}{2x-4} = 0$

$\Rightarrow f(x,y) = y(2x-4) - (x^3-3) = 0$

$= 2xy - 4y - x^3 + 3$ — ①

1) Changing x to $-x$;

$f(-x,y) = 2(-x)y - 4y - (-x)^3 + 3$

$= -2xy - 4y + x^3 + 3 \neq f(x,y)$

2) Changing y to $-y$;

$f(x,-y) = 2x(-y) - 4(-y) - x^3 + 3$

$= -2xy + 4y - x^3 + 3 \neq f(x,y)$

3) Changing x to $-x$; y to $-y$;

$f(-x,-y) = 2(-x)(-y) - 4(-y) - (-x)^3 + 3$

$= 2xy + 4y + x^3 + 3 \neq f(x,y)$

4) Changing x to y & y to x;

$f(y,x) = 2yx - 4x - y^3 + 3 \neq f(x,y)$

5) Changing x to $-y$ & y to $-x$;

$f(-y,-x) = 2(-y)(-x) - 4(-x) - (-y)^3 + 3$

$= 2xy + 4x + y^3 + 3 \neq f(x,y)$

∴ The curve is not symmetric about any line or axis.

Q10: $y = x^3 - 12x - 16.$

Soln: The given curve is $f(x,y) = y - x^3 + 12x + 16 = 0$ — ①

1) Changing x to $-x$;

$f(-x,y) = y - (-x)^3 + 12(-x) + 16$

$= y + x^3 - 12x + 16 \neq f(x,y)$

2) Changing y to $-y$;

$f(x,-y) = (-y) - x^3 + 12x + 16$

$= -y - x^3 + 12x + 16 \neq f(x,y)$

3) Changing x to $-x$ & y to $-y$;

$f(-x,-y) = -y + x^3 - 12x + 16 \neq f(x,y)$

4). Changing x to y & y to x;

$$f(y,x) = x - y^3 + 12y + 16 \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-x) - (-y)^3 + 12(-y) + 16$$
$$= -x + y^3 - 12y + 16 \neq f(x,y)$$

∴ The given curve is not symmetric about any line or axis.

Q11: $y = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$

Soln:- The given curve is $f(x,y) = 6y - (x^3 - 6x^2 + 9x + 6) = 0$

$$= 6y - x^3 + 6x^2 - 9x - 6 \quad \text{—①}$$

1) Changing x to $-x$;

$$f(-x,y) = 6y - (-x)^3 + 6(-x)^2 - 9(-x) - 6$$
$$= 6y + x^3 + 6x^2 + 9x - 6 \neq f(x,y)$$

2) Changing y to $-y$;

$$f(x,-y) = 6(-y) - x^3 + 6x^2 - 9x - 6$$
$$= -6y - x^3 + 6x^2 - 9x - 6 \neq f(x,y)$$

3) changing x to $-x$ & y to $-y$;

$$f(-x,-y) = 6(-y) - (-x)^3 + 6(-x)^2 - 9(-x) - 6$$
$$= -6y + x^3 + 6x^2 + 9x - 6 \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = 6x - y^3 + 6y^2 - 9y - 6 \neq f(x,y)$$

5) Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = 6(-x) - (-y)^3 + 6(-y)^2 - 9(-y) - 6$$
$$= -6x + y^3 + 6y + 9y - 6 \neq f(x,y)$$

∴ The given curve is not symmetric about any line & axis.

Q12: $x = (y-1)(y-2)(y-3)$

Soln:- The given curve is $x = (y-1)(y^2-5y+6)$

$$= y^3-5y^2+6y-y^2+5y-6$$

$$= y^3-6y^2+11y-6 \cancel{=0} \cancel{\text{on}}$$

$$\Rightarrow f(x,y) = y^3-6y^2+11y-6-x = 0 \quad \text{—①}$$

1) Changing x to $-x$;

$$f(-x,y) = y^3-6y^2+11y-6+x \neq f(x,y)$$

2) Changing y to $-y$;

$$f(x,-y) = (-y)^3-6(-y)^2+11(-y)-6-x$$

$$= -y^3-6y^2-11y-6-x \neq f(x,y)$$

3) Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = (-y)^3-6(-y)^2+11(-y)-6-(-x)$$

$$= -y^3-6y^2-11y-6+x \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x^3-6x^2+11x-6-y \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = \cancel{-y}\ (-x)^3-6(-x)^2+11(-x)-6-(-y)$$

$$= -x^3-6x^2-11x-6+y \neq f(x,y)$$

$\therefore$ The given curve is not symmetric about any line & axis.

Q13: $\left(\frac{x}{a}\right)^{2/3} + \left(y/b\right)^{2/3} = 1$

Soln:- The given curve is $f(x,y) = \left(\frac{x}{a}\right)^{2/3} + \left(y/b\right)^{2/3} - 1 = 0$ —①

1). Changing x to $-x$;

$$f(-x,y) = \left(-\frac{x}{a}\right)^{2/3} + (y/b)^{2/3} - 1$$

$$= (x/a)^{2/3} + (y/b)^{2/3} - 1 = f(x,y)$$

2). Changing y to $-y$;

$$f(x,-y) = (x/a)^{2/3} + (-y/b)^{2/3} - 1$$

$$= (x/a)^{2/3} + (y/b)^{2/3} - 1 = f(x,y)$$

3). Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = \left(\frac{-x}{a}\right)^{2/3} + \left(-y/b\right)^{2/3} - 1$$

$$= \left(\frac{x}{a}\right)^{2/3} + \left(y/b\right)^{2/3} - 1 = f(x,y)$$

4). changing x to y & y to x;

$$f(y,x) = (y/a)^{2/3} + (x/b)^{2/3} - 1 \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-y/a)^{2/3} + (-x/b)^{2/3} - 1$$

$$= (y/a)^{2/3} + (x/b)^{2/3} - 1 \neq f(x,y)$$

$\therefore$ The given curve is symmetric about x-axis, y-axis & origin

Q:14: $y^2 = \dfrac{x-3}{x^2-6x-7}$

Soln: The given curve is $y^2 = \dfrac{x-3}{x^2-6x-7}$

$$y^2(x^2-6x-7) = x-3$$

$$x^2y^2 - 6xy^2 - 7y^2 - x + 3 = 0$$

$$\Rightarrow f(x,y) = x^2y^2 - 6xy^2 - 7y^2 - x + 3 \quad \text{— ①}$$

1) Changing x to $-x$;

$$f(-x,y) = (-x)^2y^2 - 6(-x)y^2 - 7y^2 - (-x) + 3$$

$$= x^2y^2 + 6xy^2 - 7y^2 + x + 3 \neq f(x,y)$$

2) Changing y to $-y$;-

$$f(x,-y) = x^2(-y)^2 - 6x(-y)^2 - 7(-y)^2 - x + 3$$

$$= x^2y^2 - 6xy^2 - 7y^2 - x + 3 = f(x,y)$$

3) Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = (-x)^2(-y)^2 - 6(-x)(-y)^2 - 7(-y)^2 - (-x) + 3$$

$$= x^2y^2 + 6xy^2 - 7y + x + 3 \neq f(x,y)$$

4) changing x to y & y to x;

$$f(y,x) = y^2x^2 - 6yx^2 - 7x^2 - y + 3 \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-y)^2(-x)^2 - 6(-y)(-x)^2 - 7(-x)^2 - (-y) + 3$$

$$= y^2x^2 + 6yx^2 - 7x + y + 3 \neq f(x,y)$$

∴ The given curve is ~~not~~ symmetric about x-axis only.

Q15: $y = c \cosh \frac{x}{c}$

Soln:- The given curve is $y - c\cosh\frac{x}{c} = 0$

$$\Rightarrow f(x,y) = y - c\cosh\frac{x}{c} \quad \text{——} ①$$

1). Changing x to $-x$;

$$f(-x,y) = y - c\cosh\left(-\frac{x}{c}\right)$$

$$= y - c\cosh\frac{x}{c} = f(x,y)$$

2). Changing y to $-y$;

$$f(x,-y) = -y - c\cosh\left(\frac{x}{c}\right) \neq f(x,y)$$

3). Changing x to $-x$ & y to $-y$;

$$f(-x,-y) = -y - c\cosh\left(-\frac{x}{c}\right)$$

$$= -y - c\cosh\frac{x}{c} \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x - c\cosh\frac{y}{c} \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = -x - \cosh\left(-\frac{y}{c}\right)$$

$$= -x - \cosh\frac{y}{c} \neq f(x,y)$$

∴ The given curve is symmetric about y-axis.

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = (-y)^2(-x)^2 - 6(-y)(-x)^2 - 7(-x)^2 - (-y)+3$$
$$= y^2x^2 + 6yx^2 - 7x + y + 3 \neq f(x,y)$$

∴ The given curve is ~~not~~ symmetric about x-axis only.

Q15: $y = c\cosh\frac{x}{c}$

Soln:- The given curve is $y - c\cosh\frac{x}{c} = 0$

$$\Rightarrow f(x,y) = y - c\cosh\frac{x}{c} \quad \text{--- ①}$$

1). Changing x to $-x$;

$$f(-x,y) = y - c\cosh\left(-\frac{x}{c}\right)$$
$$= y - c\cosh\frac{x}{c} = f(x,y)$$

2). changing y to $-y$;

$$f(x,-y) = -y - c\cosh\left(\frac{x}{c}\right) \neq f(x,y)$$

3). changing x to $-x$ & y to $-y$;

$$f(-x,-y) = -y - c\cosh\left(-\frac{x}{c}\right)$$
$$= -y - c\cosh\frac{x}{c} \neq f(x,y)$$

4) Changing x to y & y to x;

$$f(y,x) = x - c\cosh\frac{y}{c} \neq f(x,y)$$

5). Changing x to $-y$ & y to $-x$;

$$f(-y,-x) = -x - \cosh\left(-\frac{y}{c}\right)$$
$$= -x - \cosh\frac{y}{c} \neq f(x,y)$$

∴ The given curve is symmetric about y-axis.

Q3 $x(x^2+y^2)=a(x^2-y^2)$

Soln:- The given curve is $x^3+xy^2-ax^2+ay^2=0$

sub $x=0 \Rightarrow 0+0-0+ay^2=0$

$$y^2=0$$

$$\boxed{y=0}$$

$\therefore$ The curve passes through origin.

Tangents at the origin are given by

$$a(x^2-y^2)=0$$

$$x^2-y^2=0$$

$$x^2=y^2 \Rightarrow y=\pm x$$

$\Rightarrow y=x$ or $y=-x$

$\Rightarrow x-y=0$ or $x+y=0$ are the tangents at origin.

Q4: $y=x+\frac{1}{x}$

Soln:- The given curve is $xy=x^2+1$

$\Rightarrow xy-x^2-1=0$

sub. $x=0 \Rightarrow 0-0-1=0$

$\Rightarrow -1=0$; which is not possible

$\therefore$ The curve does not passes through origin.

Q5: $y^2(a-x)=x^3$; $a>0$

Soln:- sub $x=0 \Rightarrow y^2(a-0)-0=0$

$\Rightarrow y^2=0$

$\Rightarrow y=0.$

$\therefore$ The curve passes through origin.

Tangents at origin are given by

$$y^2a=0$$

$$y^2=0 \Rightarrow y=0$$

$\therefore y=0$ is the tangent at origin

i.e. x-axis is the tangent at origin.

Q6: $y = \frac{x^2+1}{x+1}$

Soln:- The given curve is $y(x+1) = x^2+1$

$$xy + y - x^2 - 1 = 0 \quad — ①$$

substitute $x=0 \Rightarrow 0 + y - 0 - 1 = 0$

$$y = 1$$

$\therefore$ curve does not pass through origin.

Q7: $y = \frac{x^3+1}{x}$.

Soln:- The given curve is $xy = x^3+1$

$$xy - x^3 - 1 = 0 \quad — ①$$

sub $x=0 \Rightarrow y\ 0 - 0 - 1 = 0$; which is not true.

$\therefore$ The curve does not passes through origin.

Q8: $y = \frac{x^3}{1+x^2}$

Soln:- The given curve is $y(1+x^2) = x^3$

$$\Rightarrow x^3 - y - x^2 y = 0$$

substitute $x=0 \Rightarrow 0 - y - 0 = 0$

$$\Rightarrow \boxed{y=0}$$

$\therefore$ The curve passes through origin

Tangents at origin are given by

$$y=0$$

$\therefore$ x-axis is the tangent at origin.

Q9: $y = \frac{x^3-3}{2x-4}$

The given curve is $(2x-4)y = x^3-3$

$$\Rightarrow x^3 - 3 - 2xy + 4y = 0$$

sub $x=0$; $\Rightarrow 0 - 3 - 0 + 4y = 0$

$$4y = 3 \Rightarrow y = 3/4$$

$\therefore$ Curve does not passes through origin

Q10: $y = x^3 - 12x - 16.$

Soln:- The given curve is $x^3 - 12x - y - 16 = 0$

sub $x = 0 \Rightarrow 0 - 0 - y - 16 = 0$

$y = -16$

$\therefore$ Curve does not passes through origin.

Q11: $y = \frac{1}{6}(x^3 - 6x^2 + 9x + 6).$

Soln:- The given curve is $6y = x^3 - 6x^2 + 9x + 6$

sub $x = 0 \Rightarrow 6y = 6 \Rightarrow \boxed{y = 1}$

$\therefore$ The curve does not passes through origin.

Q12: $x = (y-1)(y-2)(y-3)$

Soln: sub $x = 0 \Rightarrow (y-1)(y-2)(y-3) = 0$

$\Rightarrow y = 1, 2, 3.$

$\therefore$ The curve does not passes through origin.

Q13: $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

Soln:- sub $x = 0 \Rightarrow 0 + (y/b)^{2/3} = 1$

$\Rightarrow y/b = 1 \Rightarrow \boxed{y = b}$

$\therefore$ The curve does not passes through origin.

Q14: $y^2 = \frac{x-3}{x^2 - 6x - 7}$

Soln:- The given curve is $x^2y^2 - 6xy^2 - 7y^2 = x - 3$

sub $x = 0 \Rightarrow 0 - 0 - 7y^2 = 0 - 3$

$\Rightarrow y^2 = 3/7$

$\therefore$ curve does not passes through origin.

Q15: $y = c \cosh x/c$

sub $x = 0 \Rightarrow \boxed{y = c}$

$\therefore$ The curve does not passes through origin.

DOMAIN & RANGE

If the curve is in the form $y = f(x)$

$\therefore\ D_f = \{x : f(x) \text{ is defined}\}.$

& $R_f = \{y = f(x) ; x \in D_f\}$

Q1: $x^3 + y^3 = 3axy\ ;\ a \geq 0$

Soln: The given curve is $x^3+y^3 = 3axy$ — ①

from the above eqn; $D_f = \{x : \{ x \in R\}$

& $R_f = \{y : y \in R\}$

but here, x & y both can't be negative

$\because$ L.H.S $= (-ve)^3 + (-ve)^3 = (-ve) + (-ve) = -ve$

R.H.S $= 3a(-ve)(-ve) = 3a(+ve)$

$\Rightarrow$ L.H.S. $\neq$ R.H.S

$\therefore$ No portion of the curve lies in 3rd quadrant.

Q2: $a^2y^2 = x^2(a^2 - x^2)$

Soln:- The given curve is $y^2 = \dfrac{x^2(a^2-x^2)}{a^2}$ — ①

Here, y is defined iff ~~$x\ a^2 - x^2 \geq 0$~~

$x^2 < a^2$

$\Rightarrow |x| < a$

$\Rightarrow -a < x < a$

$\therefore\ D_f = (-a, a)$

$R_f = R - \{-a, a\}$

Q3: $x(x^2+y^2) = a(x^2 - y^2)$

Soln:- The given curve can be written as

$$xy^2 + x^3 = ax^2 - ay^2$$

$$xy^2 + ay^2 = ax^2 - x^3$$

$$y^2 = \frac{ax^2 - x^3}{x+a} \quad \text{—— ①}$$

$$y^2 = \frac{x^2(a-x)}{x+a}$$

$$y = \pm x\sqrt{\frac{a-x}{a+x}} \quad \text{— ②}$$

$\therefore$ y will exist if $a-x>0$ & $a+x>0$

if $a>x$ & $x>-a$

$\Rightarrow \quad -a<x<a$

$\therefore \; D_f = (-a, a)$

$R_f = R - \{-a\}$

Q4: $y = x + \frac{1}{x}$

Soln:- here y is not defined for $x=0$

$\therefore \; D_f = R - \{0\}$

Also; $xy = x^2+1$

$x^2 - xy + 1 = 0$

$$x = \frac{-(-y) \pm \sqrt{(-y)^2 - 4\times1\times1}}{2\times1}$$

$$x = \frac{y \pm \sqrt{y^2-4}}{2}$$

$\therefore$ x is defined $\forall \; y^2-4 \geq 0$

$y^2 \geq 4$

$|y| \geq 2$

$-2 \geq y \geq 2$

$\therefore \; R_f = (-\infty, -2] \cup [2, \infty)$

Q5: $y^2(a-x) = x^3$; $a>0$

Soln:-

$$y^2 = \frac{x^3}{a-x}$$

$$y = \pm\sqrt{\frac{x^3}{a-x}}.$$

here, y is defined iff $\frac{x^3}{a-x} \geqslant 0$

when $x>0 \Rightarrow \frac{x^3}{a-x} \geqslant 0$

$\Rightarrow a-x>0$

$\Rightarrow x<a \Rightarrow \boxed{a>x>0}$

when $x<0 \Rightarrow \frac{x^3}{a-x} \geqslant 0$

$\Rightarrow a-x<0$

$\Rightarrow x>a \Rightarrow \boxed{0>x>a}$

which is a contradiction ∵ given $a>0$

$\therefore D_f = [0,a)$

& R_f = set of all real no's.

Q6: $y = \frac{x^2+1}{x+1}$

Soln:- Here, y is defined for all ~~ex~~ x except $x+1=0$ ie $x=-1$

$\therefore D_f = R - \{-1\}$

for Range;

$$xy + y = x^2+1$$

$$x^2 - xy + (y-1) = 0$$

$$x = \frac{y \pm \sqrt{y^2 - 4(1-y)}}{2}$$

$$= \frac{y \pm \sqrt{y^2+4y-4}}{2}$$

$$= \frac{y \pm \sqrt{(y+2)^2-8}}{2}$$

$\therefore$ x exists iff $(y+2)^2-8 \geq 0$

$$(y+2)^2 \geq 8$$

$$|y+2| \geq 2\sqrt{2}$$

$$\Rightarrow -2\sqrt{2} \geq (y+2) \geq 2\sqrt{2}$$

$$\Rightarrow -2\sqrt{2}-2 \geq y+2-2 \geq 2\sqrt{2}-2$$

$$\Rightarrow -2\sqrt{2}-2 \geq y \geq 2\sqrt{2}-2$$

$$\therefore y \in (-\infty, -2\sqrt{2}-2] \cup [2\sqrt{2}-2, \infty)$$

$\therefore$ $D_f = R - \{-1\}$

$R_f = (-\infty, -2\sqrt{2}-2] \cup [2\sqrt{2}-2, \infty)$

Q:7: $y = \dfrac{x^3+1}{x}$

Soln: Here y is defined except $x \neq 0$

$\therefore$ $D_f = R - \{0\}$

Q8: $y = \dfrac{x^3}{1+x^2}$

Soln: Here $D_f = R$ $\left(\because 1+x^2 \neq 0\right.$

$\left. x^2 \neq -1 \text{; which does not exists}\right)$

Q9: $y = \dfrac{x^3-3}{2x-4}$

Soln y is defined for all except $2x-4=0$

$x = 2$

$\therefore$ $D_f = R - \{2\}$

Also; curve does not lies in 3rd quadrant

$$= \frac{y \pm \sqrt{y^2+4y-4}}{2}$$

$$= \frac{y \pm \sqrt{(y+2)^2-8}}{2}$$

$\therefore$ x exists iff $(y+2)^2-8 \geq 0$

$(y+2)^2 \geq 8$

$|y+2| \geq 2\sqrt{2}$

$\Rightarrow -2\sqrt{2} \geq (y+2) \geq 2\sqrt{2}$

$\Rightarrow -2\sqrt{2}-2 \geq y+2-2 \geq 2\sqrt{2}-2$

$\Rightarrow -2\sqrt{2}-2 \geq y \geq 2\sqrt{2}-2$

$\therefore y \in (-\infty, -2\sqrt{2}-2] \cup [2\sqrt{2}-2, \infty)$

$\therefore$ $D_f = R - \{-1\}$

$R_f = (-\infty, -2\sqrt{2}-2] \cup [2\sqrt{2}-2, \infty)$

Q:7: $y = \frac{x^3+1}{x}$

Soln: Here y is defined except $x \neq 0$

$\therefore D_f = R - \{0\}$

Q8: $y = \frac{x^3}{1+x^2}$

Soln: Here $D_f = R$ $\left(\because 1+x^2 \neq 0,\ x^2 \neq -1\text{; which does not exists}\right)$

Q9: $y = \frac{x^3-3}{2x-4}$

Soln: y is defined for all except $2x-4=0$

$x=2$

$\therefore D_f = R - \{2\}$

Also; curve does not lies in 3rd quadrant

Q14: $y^2 = \frac{x-3}{x^2-6x-7}$.

Soln:- $y^2 = \frac{x-3}{x^2-7x+x-7} = \frac{x-3}{x(x-7)+1(x-7)}$

$= \frac{x-3}{(x-7)(x+1)}$

$\therefore$ y is defined for all values of x except $x=-1 \& 7$
& when $x>3, x<7$

$\therefore D_f = (-\infty,-1) \cup (3,7)$

Range; $x^2y^2 - 6xy^2 - 7y^2 = x-3$

$x^2y^2 - (6y^2+1)x - (7y^2-3) = 0$

$$x = \frac{(6y^2+1) \pm \sqrt{(6y^2+1)^2 - 4y^2 x - (7y^2-3)}}{2y^2}$$

$$x = \frac{(6y^2+1) \pm \sqrt{(6y^2+1)^2 + 4y^2(7y^2-3)}}{2y^2}$$

$\therefore$ x is defined when $(6y^2+1)^2 + 4y^2(7y^2-3) \geq 0$

$36y^4 + 1 + \cancel{12}y^2 + 28y^4 - \cancel{12}y^2 \geq 0$

$64y^4 + 1 \geq 0$

$\therefore$ x is defined for all values of y

$\therefore R_f = R$

Q15: $y = c \cosh \frac{x}{a}$

Soln: y is defined for all real values of x.

$\therefore D_f = R$

But R_f = only +ve values of 'y'.

POINTS OF INTERSECTION

To find the points of intersection of the curve with the

1) x-axis, sub $y=0$
2) y-axis, sub $x=0$
3) line of symmetry; sub $x=y$.

Q1: $x^3+y^3=3axy$; $a\geq 0$

Soln:- The curve meets x-axis at; $y=0$

$$\Rightarrow x^3+0=0$$
$$\Rightarrow x=0$$

The curve meets y-axis at; $x=0$

$$\Rightarrow 0+y^3=0 \Rightarrow y=0$$

The curve meets (intersects) $y=x$

$$x^3+x^3=3ax^2$$
$$2x^3-3ax^2=0$$
$$x^2(2x-3a)=0$$
$$x^2=0;\ x=\frac{3a}{2}$$

$\therefore$ The points of Intersection are $(0,0)$ & $\left(\frac{3a}{2},\frac{3a}{2}\right)$.

Q2: $a^2y^2=x^2(a^2-x^2)$

Soln:- The curve meets x-axis at; $y=0$

$$① \Rightarrow 0=x^2(a^2-x^2)$$
$$\Rightarrow x^2=0\ ;\ a^2-x^2=0$$
$$\Rightarrow x=0;\ x=\pm a$$

The curve meets y-axis at; $x=0$

$$① \Rightarrow a^2y^2=0 \Rightarrow y=0$$

The curve intersects $y=x$ line at

$$① \Rightarrow a^2x^2=x^2(a^2-x^2)$$
$$a^2x^2=a^2x^2-x^4$$

$\Rightarrow \quad x^4 = 0$

$\Rightarrow \quad x = 0$

$\therefore$ The curve intersects x-axis at $(a,0)$, $(-a,0)$ & symmetric line, y-axis at $(0,0)$.

Q3: $x(x^2+y^2) = a(x^2-y^2)$ — ①

Soln: The curve intersects x-axis at $y=0$;

① $\Rightarrow \quad x(x^2+0) = a(x^2-0)$

$$x^3 = ax^2 \Rightarrow x^3 - ax^2 = 0$$

$$x^2(x-a) = 0$$

$$x = 0;\ x = a$$

The curve intersects y-axis at $x=0$;

① $\Rightarrow \quad 0(0+y^2) = a(0^2-y^2)$

$$\cancel{y} \quad 0 = ay^2 \Rightarrow \boxed{y=0}$$

The curve intersects line of symmetry at $x=y$

① $\Rightarrow \quad x(x^2+x^2) = a(x^2-x^2)$

$$x(2x^2) = 0 \Rightarrow \boxed{x=0}$$

$\therefore$ The curve intersects x-axis at $(0,0)$ & $(a,0)$; intersects y-axis & line of symmetry at $(0,0)$.

Q4: $y = x + \frac{1}{x}$

Soln:- The given curve is $xy = x^2+1$ — ①

The curve intersects x-axis at $y=0$;

① $\Rightarrow \quad 0 = x^2+1 \Rightarrow x^2 = -1$; which is not possible

$\therefore$ curve does not intersect x-axis.

The curve intersects y-axis at $x=0$;

$0 = 0+1$; which is not possible

$\therefore$ Curve does not intersect y-axis.

The curve intersects line of symmetry at $y=x$

① ⇒ $x^2 = x^2+1$

$0 = 1$; which is not possible

∴ The given curve neither meets any co-ordinate axis nor the line of symmetry.

Q5: $y^2(a-x) = x^3$; $a>0$ ——①

Soln:- The given curve meets x-axis at $y=0$;

① ⇒ $0 = x^3 \Rightarrow x=0$

The given curve meets y-axis at $x=0$;

① ⇒ $y^2(a-0) = 0$

$\Rightarrow y=0$

The given curve meets line of symmetry at $x=y$

$$x^2(a-x) = x^3$$
$$ax^2 - x^3 = x^3$$
$$ax^2 - 2x^3 = 0$$
$$x^2(a-2x) = 0$$
$$x=0;\ x=a/2$$

∴ The curve meets x-axis at (0,0), y-axis at (0,0) & line of symmetry at $(a/2, a/2)$.

Q6: $y = \frac{x^2+1}{x+1}$

Soln:- The given curve is $y(x+1) = x^2+1$. ——①

The curve ①, meets x-axis at $y=0$;

$0 = x^2+1 \Rightarrow x^2=-1$, which is not possible

The curve ①, meets y-axis at $x=0$

~~$x=0$~~ $y(0+1) = 0+1$

$y=1$

The curve meets line of symmetry at $x=y$;

$$① \Rightarrow \quad x(x+1) = x^2+1$$
$$x^2+x = x^2+1$$
$$x = 1.$$

$\therefore$ The given curve meets y-axis at (1,1) & line of symm at (1,1) but does not touch/intersect x-axis.

Q7: $y = \frac{x^3+1}{x}$

Soln:- The given curve is $xy = x^3+1$ — ①

The curve ① meets x-axis at $y=0$;
$$0 = x^3+1$$
$\Rightarrow x=-1$ satisfies above eqn

The curve ① meets y-axis at $x=0$;
$$0 = 0+1$$; which is not possible

The curve ① meets line of symmetry at $y=x$
$$① \Rightarrow \quad x^2 = x^3+1$$
$$\Rightarrow \quad x^3-x^2+1=0$$; which does not have real soln.

$\therefore$ The given curve meets x-axis at (-1,0) but does not touch y-axis & line of symmetry.

Q8: $y = \frac{x^3}{1+x^2}$

Soln:- The given curve can be written as $y(1+x^2) = x^3$ — ①

① meets x-axis at $y=0$;
$$① \Rightarrow \quad 0 = x^3$$
$$\Rightarrow \boxed{x=0}$$

① meets y-axis at $x=0$;
$$y(1+0) = 0 \Rightarrow \boxed{y=0}$$

① meets line of symmetry at $x=y$
$$x(1+x^2) = x^3$$

$$x + x^3 = x^3$$

$$\Rightarrow \boxed{x=0}$$

∴ The given curve meets x-axis, y-axis & line of symmetry at origin.

Q:9: $y = \dfrac{x^3-3}{2x-4}$

Soln:- The given curve can be written as

$$y(2x-4) = x^3-3.$$

$$2xy - 4y = x^3-3 \quad — ①$$

The curve ①, meets x-axis at $y=0$

$$0 - 0 = x^2-3$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow \boxed{x = \pm\sqrt{3}}$$

The curve ①, meets y-axis at $x=0$;

$$0 - 4y = 0-3$$

$$\Rightarrow 4y = 3 \Rightarrow \boxed{y = 3/4}$$

The curve ①, meets line of symmetry at $x=y$,

$$2x^2 - 4x = x^3-3$$

$$\Rightarrow x^3 - 2x^2 + 4x - 3 = 0$$

$x=1$, ~~one~~ satisfies the above eqn.

∴ The given curve meets x-axis at $(\sqrt{3},0)$, $(-\sqrt{3},0)$, y-axis at $(0,3/4)$ & line of symmetry at $(1,1)$.

Q10: $y = x^3-12x-16$

The curve meets x-axis at $y=0$

$$\Rightarrow 0 = x^3-12x-16$$

$\Rightarrow$ $x=4$ satisfies above eqn.

$$\Rightarrow (x-4)(x^2+4x+4)=0$$

$$(x-4)(x+2)^2=0$$

$$x = 4, -2, -2$$

$$\begin{array}{r} x^2+4x+4 \\ x-4 \overline{) x^3-12x-16} \\ \underset{-}{x^3}\ \underset{+}{-4x^2} \\ \hline 4x^2-12x-16 \\ \underset{-}{4x^2}\ \underset{+}{-16x} \\ \hline 4x-16 \\ 4x-16 \\ \hline 0 \end{array}$$

The curve meets y-axis at $x=0$;

$$y = 0-0-16$$

$$\Rightarrow \boxed{y=-16}$$

The curve meets line of symmetry at $x=y$

$$x = x^3-12x-16$$

$$x^3-13x-16=0$$

The curve meets x-axis at $(-2,0)$, $(4,0)$, y-axis at $(0,-16)$.

Q11: $y = \frac{1}{6}(x^3-6x^2+9x+6)$

Soln:- The given curve can be written as

$$6y = x^3-6x^2+9x+6 \quad — ①$$

The curve ①, meets x-axis at $y=0$

$$x^3-6x^2+9x+6=0$$

~~$x=-1$ satisfies above eqn~~

x does not satisfies above for any real value

$$x+1\,)\,x^3-6x^2+9x+6\,(\,x^2-7x$$
$$\underline{-x^3\ \ \underset{-}{+}x^2}$$
$$-7x^2+9x+6$$
$$-7x^2-7x$$

The curve ①, meets y-axis at $x=0$

$$y = \frac{1}{6}(0-0+0+6) = 1$$

$\therefore$ curve meets y-axis at $(0,1)$.

Q12: $x = (y-1)(y-2)(y-3)$

Soln:- The curve meets x-axis at $y=0$

$$\Rightarrow \quad x = (-1)(-2)(-3) = -6$$

The curve meets y-axis at $x=0$,

$$0 = (y-1)(y-2)(y-3)$$

$$\Rightarrow y = 1, 2, 3.$$

The curve meets the line of symmetry $y=x$ at

$$y = (y-1)(y-2)(y-3)$$

It does not satisfy.

$\therefore$ The curve meets x-axis at $(-6,0)$

y-axis at $(0,1)$, $(0,2)$, $(0,3)$.

Q: $y^2 = \frac{x-3}{x^2-6x-7}$.

Soln:- The given curve can be rewritten as

$$y^2 = \frac{(x-3)}{x^2-7x+x-7} = \frac{x-3}{x(x-7)+1(x-7)} = \frac{x-3}{(x-7)(x+1)} \quad -①$$

The above eqn ① intersects x-axis at $y=0$;

① ⇒ $x-3=0 \Rightarrow x=3$.

The above eqn ① intersects y-axis at $x=0$,

$$y^2 = \frac{-3}{-7} \Rightarrow y = \pm\sqrt{\frac{3}{7}}$$

∴ The curve meets x-axis at $(3,0)$, y-axis at $\left(0, \frac{\sqrt{3}}{\sqrt{7}}\right)$ & $\left(0, -\frac{\sqrt{3}}{\sqrt{7}}\right)$

$\left(\frac{\sqrt{3}}{\sqrt{7}} = 0.65\right)$

Q: $y = c \cosh \frac{x}{c}$

Soln:- The given curve intersects x-axis at $y=0$,

$\Rightarrow \quad 0 = c \cosh \frac{x}{c}$

$\Rightarrow c \neq 0; \ \cosh \frac{x}{c} = 0$;

∴ Curve does not intersect x-axis.

The curve intersects y-axis at $x=0$

$y = c \cdot \cosh 0$

$y = c$

∴ Curve intersects y-axis at $(0, c)$